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Fuzzy Bigroup from another Viewpoint

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Abstract

In group theory, given two groups G and H, it is possible to construct a new group from the Cartesian product of G and H, $G \times H$. With this as a motivation, we replicate this concept in fuzzy group algebra. In this paper, we take a slight deviation from the familiar definition of fuzzy bigroup by looking at fuzzy bigroup from the idea of Cartesian product of groups. We define Cartesian fuzzy function on groups and give examples. We also define Cartesian fuzzy bigroup and study some of its basic properties.

Keywords: Bigroups, Cartesian fuzzy function, Fuzzy bigroups.

Introduction

Zadeh (1965) introduced fuzzy sets. Since then, this idea has been extended to many branches of mathematics. Fuzzy subsystems of algebraic structures have been introduced and extensively studied by several mathematicians. In particular, in the areas of fuzzy group theory, Rosenfeld (1971) introduced the notion of fuzzy subgroups. He gave the conditions for a fuzzy subsets of a non-empty set to be a fuzzy subgroup and discussed its characterization.

Maggu (1994) was the first to introduce the notion of bigroup. He defined a bigroup and investigated whether or not those properties satisfied by groups are also satisfied by bigroups. These notions were further studied by (Vasantha and Meiyappan, 2003). These authors gave modifications of some results earlier proved by Maggu. Among these results were characterization theorems for sub-bigroup. Meiyappan et al. (1998) introduced the notion of fuzzy union of any two fuzzy subsets of two

distinct sets. They used this idea to define fuzzy sub-bigroup of a bigroup and studied its properties. They further established the relationship between fuzzy subgroup of a group and fuzzy sub-bigroup of a bigroup.

Akinola and Agboola (2012) pointed out the lapses observed in the fuzzy bigroup defined by Meiyappan et al. (1998). According to Meiyappan, if $G = (G_1 \cup G_2, +, \circ)$ is a bigroup, then $\gamma \colon G \rightarrow [0,1]$ is said to be a fuzzy sub-bigroup of the bigroup G if there exist two fuzzy subsets Y_1 (of G_1) and Y_2 (of G_2) such that: $(Y_1, +)$ is a fuzzy subgroup of $(G_1, +)$, (Y_2, \cdot) is a fuzzy subgroup of G_2 , $(Y_1, +)$, and $(Y_2, -)$ is a fuzzy subgroup of $(Y_1, +)$, and $(Y_2, -)$ in the definition look as if "+" and "·"are operations defined on the fuzzy

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sets γ_1 and γ_2 respectively. In this paper, we take a slight deviation from the familiar definition of fuzzy bigroup by looking at fuzzy bigroup from the idea of Cartesian product of sets (groups). We define Cartesian fuzzy function on groups and give examples. We also give conditions to be satisfied for Cartesian fuzzy function to be a fuzzy bigroup and study some of its fundamental properties.

Preliminaries

We present in this section some of the results we found useful in establishing our results in the next section. For details, please see the references.

Note that fuzzy subgroup and fuzzy group are used interchangeably likewise fuzzy sub bigroup and fuzzy bigroup just like fuzzy set and fuzzy subset.

Definition (2.1): A set $(G, +, \cdot)$ with two binary operations "+" and "·" is called a bi-group if there exist two proper subsets G_1 and G_2 of G such that

- $(i) G = G_1 \cup G_2,$
- (ii) $(G_1, +)$ is a group,
- (iii) (G_2,\cdot) is a group.

Definition (2.2): A subset $H(\neq \phi)$ of a bi-group $(G,+,\cdot)$ is called a sub bi-group of G if H itself is a bi-group under the operations of "+" and "·" defined on G.

Theorem (2.3): Let $(G, +, \cdot)$ be a bi-group. If the subset $H(\neq \phi)$ of a bi-group G is a sub bi-group of G, then (H, +) and (H, \cdot) are generally not groups.

Definition (2.4): Let G be a non empty set. A mapping $\gamma\colon G\to [0,1]$ is called a fuzzy subset of G. **Definition (2.5):** Let γ be a fuzzy set in a set G.

Then, the t-level subset of γ , denoted by γ_t is defined as: $\gamma_t = \{x \in G: \gamma(x) \ge t\}$ for $t \in [0,1]$.

Definition (2.6): Let γ be a fuzzy set in a group G. Then, γ is said to be a fuzzy subgroup of G, if the following hold:

- (i) $\gamma(xy) \ge \min\{\gamma(x), \gamma(y)\} \ \forall \ x, y \in G;$
- (ii) $\gamma(x^{-1}) = \gamma(x) \ \forall \ x \in G.$

Proposition (2.7): Let G be a group with the unit element e and γ be a fuzzy subgroup of G. For all $x,y\in G$,

- (i) $\gamma(x) \leq \gamma(e)$,
- (ii) $\gamma(x) = \gamma(x^{-1}),$
- (iii) $\gamma(xy^{-1}) = \gamma(e) \Rightarrow \gamma(x) = \gamma(y)$.

Definition (2.8): Let γ be a fuzzy subgroup of G. Then, the level subset γ_t , for $t \in Im \gamma$ is a subgroup of G and is called the level subgroup of G.

Definition (2.9): Let γ_1 be a fuzzy subset of a set X_1 and γ_2 be a fuzzy subset of a set X_2 , then the fuzzy union of the sets γ_1 and γ_2 is defined as a function $\gamma_1 \cup \gamma_2 \colon X_1 \cup X_2 \to [0,1]$ given by:

$$(\gamma_1 \cup \gamma_2)(x) = \begin{cases} \max(\gamma_1(x), \gamma_2(x)) & \text{if } x \in X_1 \cap X_2, \\ \gamma_1(x) & \text{if } x \in X_1 \text{ and } x \notin X_2, \\ \gamma_2(x) & \text{if } x \in X_2 \text{ and } x \notin X_1. \end{cases}$$

Definition (2.10): Let $G = (G_1 \cup G_2, +, \cdot)$ be a bigroup. Then $\mu: G \to [0,1]$ is said to be a fuzzy subbigroup of the bigroup G if there exist two fuzzy subsets $\mu_1(\text{of }G_1)$ and $\mu_2(\text{of }G_2)$ such that:

- (i) $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$,
- (ii) (μ_2, \cdot) is a fuzzy subgroup of (G_2, \cdot) , and
- (iii) $\mu = \mu_1 \cup \mu_2.$

The Results

The notations used in this section are conventional. \times represents both the usual product of real numbers and Cartesian product of sets. In each case, the meaning is always clear from the context. Λ stands for minimum as widely used in lattice theory. Let (G,*) and (H,*) be groups. Let $\phi,\psi:G,H\to(0,1)$ be fuzzy subgroups of G and H respectively. For any element $g_i\in G$, let $\phi(g_i)$ be a membership grade assigned to $g_i\in G$ in (0,1). Similarly $\psi(h_j)$ denotes the membership grade of element $h_j\in H$ in(0,1).

Definition (3.1): Let $\eta:G\times H\to (0,1)$ be defined by $\eta(g,h)=\phi(g)\times \psi(h)$ for $g\in G$ and $h\in H$, η is called a Cartesian fuzzy function of the fuzzy groups ϕ and ψ .

 $\eta(h,g): H \times G \rightarrow (0,1)$ is defined in a similar way.

Proposition (3.2): For any groups G and H, $\eta(g,h) = \eta(h,g)$.

Proof: This is obvious since $\phi(g) \times \psi(h) = \psi(h) \times \phi(g)$ for $g \in G$ and $h \in H$ where $\phi(g), \psi(h) \in (0,1)$.

Example (3.3): Let (G,*) and (H, \circ) be groups as shown in the following Cayley table;

Cayley tables of (G,*) and $(H \circ)$

G	i	1	m	n	o	р
I		I	m	n	0	р
ı	_	m	i	р	n	0
m	m	i	_	0	р	n
n	n	0	Р	i	1	m
0	0	p	n	m	i	1

Define a fuzzy set $\phi: G \to (0,1)$ by

$$\phi(x) = \begin{cases} 0.8 & \text{if } x = i, \\ 0.6 & \text{if } x \in \{o, p\}, \\ 0.5 & \text{if } x \in \{l, m, n\} \end{cases}$$

and another fuzzy set $\psi: H \to (0,1)$ by: $\psi(e) = 1$, $\psi(a) = 0.5$, $\psi(c) = 0.3$.

It is easy to see that both ϕ and ψ are fuzzy subgroups of groups G and H respectively.

The Cartesian products of the groups G and H is given as;

$$G \times H = \{(i, e), (i, a), (i, b), (i, c), (l, e), (l, a),$$

$$(l, b), (l, c), (m, e), (m, a), (m, b), (m, c)\}$$

By definition (3.1), $\eta:G\times H\to (0,1)$ are given as; {0.80, 0.40, 0.24, 0.24, 0.5, 0.25, 0.15, 0.15, 0.5, 0.25, 0.15, 0.15, 0.15, 0.15, 0.25, 0.15, 0.42, 0.30, 0.18, 0.18, 0.6, 0.30, 0.24, 0.24}

Proposition (3.4): For any groups G and H, $\eta(e_1,e_2) \geq \eta(g,h)$, $g \in G$ and $h \in H$ where e_1 is the identity element of G and e_2 is the identity element of H.

Proof: Let $\phi:G \to (0,1)$ and $\psi:G \to (0,1)$ be fuzzy subgroups of groups G and H respectively. $\eta(e_1,e_2)=\phi(e_1)\times \psi(e_2)$. By proposition (2.7), for any $g\in G$, $\phi(e_G)\geq \phi(g)$ and for any $h\in H$, $\psi(e_H)\geq \phi(h)$. Therefore, for $g,h\in G\times H$, $\phi(e_1)\times \psi(e_2)\geq \phi(g)\times \psi(h)$. Hence $\eta(e_1,e_2)\geq \eta(g,h)$

Definition (3.5): The Cartesian fuzzy function $\eta: G \times H \to (0,1)$ is said to be a fuzzy bigroup if

$$\begin{array}{ll} 1, & \eta(g,h) \geq \Lambda\{\varnothing(g_1), \varnothing(g_2)\} \times \\ \Lambda\{\psi(h_1), \psi(h_2)\} & \text{for every} & g = g_1g_2, & h = h_1h_2 \\ \text{in G and H} \, . \end{array}$$

2.
$$\eta(g,h) \ge \eta(g^{-1},h^{-1}).$$

In sequel, we denote the element (x, y) of the cartesian product of X and Y by xy.

0	e	а	Ь	С
e	e	а	۵	U
α	a	е	U	Ь
Ь	Ь	С	e	α
С	С	Ь	α	e

Example (3.6): Let ϕ and $\psi:G,H\to(0,1)$ be fuzzy subgroups of (G,*) and (H,*) respectively as defined in example(3.3). From the same example, $\eta(m,b)=0.5$.

From the table,

$$\begin{split} \Lambda\{\emptyset(i),\emptyset(m)\} &\times \Lambda\{\psi(e),\psi(l)\} \\ &= \Lambda\{\emptyset(i),\emptyset(m)\} \times \Lambda\{\psi(a),\psi(e)\} \\ &= \Lambda\{\emptyset(i),\emptyset(m)\} \times \Lambda\{\psi(b),\psi(e)\} \\ &= \Lambda\{\emptyset(l),\emptyset(l)\} \times \Lambda\{\psi(e),\psi(b)\} \\ &= \Lambda\{\emptyset(l),\emptyset(l)\} \times \Lambda\{\psi(a),\psi(c)\} \\ &= \Lambda\{\emptyset(m),\emptyset(i)\} \times \Lambda\{\psi(e),\psi(b)\} \\ &= \Lambda\{\emptyset(m),\emptyset(i)\} \times \Lambda\{\psi(e),\psi(b)\} \\ &= \Lambda\{\emptyset(m),\emptyset(i)\} \times \Lambda\{\psi(b),\psi(e)\} \\ &= \Lambda\{\emptyset(m),\emptyset(i)\} \times \Lambda\{\psi(e),\psi(a)\} \\ &= \Lambda\{\emptyset(m),\emptyset(i)\} \times \Lambda\{\psi(c),\psi(a)\} \\ &= \Lambda\{\emptyset(n),\emptyset(p)\} \times \Lambda\{\psi(e),\psi(b)\} \\ &= \Lambda\{\emptyset(n),\emptyset(p)\} \times \Lambda\{\psi(b),\psi(c)\} \\ &= \Lambda\{\emptyset(n),\emptyset(p)\} \times \Lambda\{\psi(b),\psi(c)\} \\ &= \Lambda\{\emptyset(o),\emptyset(n)\} \times \Lambda\{\psi(c),\psi(b)\} \\ &= \Lambda\{\emptyset(o),\emptyset(n)\} \times \Lambda\{\psi(c),\psi(a)\} \\ &= \Lambda\{\emptyset(o),\emptyset(a)\} \times \Lambda\{\psi(a),\psi(a)\} \\ &= \Lambda\{\emptyset(o),\emptyset(a)\} \times \Lambda\{\psi(a),\psi(a)\} \\ &= \Lambda\{\emptyset(a),\emptyset(a)\} \times \Lambda\{\psi(a),\psi(a)\}$$

= 0.18

*,0	ie	ia	ib	ic	le	la	lb	lc	me	ma	Mb	mc	ne	na	nb	nc	oe	oa	ob	0C	pe	pa	pb	рс
ie	ie	ia	ib	ic	le	la	lb	lc	me	ma	Mb	mc	ne	na	nb	nc	oe	oa	ob	0C	pe	pa	pb	рс
ia	ia	ie	ic	ib	la	le	lc	lb	ma	me	Mc	mb	na	ne	nc	nb	oa	oe	0C	ob	pa	pe	рс	Pb
ib	ib	ic	ie	ia	lb	lc	le	la	mb	mc	Me	ma	nb	nc	ne	na	ob	oc	oe	oa	pb	рс	pe	Pa
ic	ic	ib	ia	ie	lc	lb	la	le	mc	mb	Ma	me	nc	nb	na	ne	0C	ob	oa	oe	рс	pb	pa	pe
le	le	la	lb	lc	me	ma	mb	mc	ie	ia	Ib	ic	pe	pa	pb	pc	ne	na	nb	nc	oe	oa	ob	0C
la	la	le	lc	lb	ma	me	mc	mb	ia	ie	ic	ib	pa	pe	pc	pb	na	ne	nc	nb	oa	oe	OC.	ob
lb	lb	lc	le	la	mb	mc	me	ma	ib	ic	ie	ia	pb	pc	pe	pa	nb	nc	ne	na	ob	oc	oe	oa
lc	lc	lb	la	le	mc	mb	ma	me	ic	ib	ia	ie	pc	pb	pa	pe	nc	nb	na	ne	0C	ob	oa	oe
me	me	ma	mb	mc	ie	ia	ib	ic	le	la	lb	lc	oe	oa	ob	OC.	pe	pa	pb	pc	ne	na	nb	nc
ma	ma	me	mc	mb	ia	ie	ic	ib	la	le	lc	lb	oa	oe	0C	ob	pa	pe	pc	pb	na	ne	nc	nb
mb	mb	mc	me	ma	ib	ic	ie	ia	lb	lc	le	la	ob	0C	oe	oa	pb	pc	pe	pa	nb	nc	ne	na
mc	mc	mb	ma	me	ic	ib	ia	ie	lc	lb	la	le	OC.	ob	oa	oe	pc	pb	pa	pe	nc	nb	na	ne
ne	ne	na	nb	nc	oe	oa	ob	00	pe	pa	pb	рс	ie	ia	ib	ic	le	la	lb	lc	me	ma	mb	mc
na	na	ne	nc	nb	oa	oe	OC.	ob	pa	pe	pc	pb	ia	ie	ic	ib	la	le	lc	lb	ma	me	mc	mb
nb	nb	nc	ne	na	ob	0C	oe	oa	pb	pc	pe	pa	ib	ic	ie	ia	lb	lc	le	la	mb	mc	me	ma
nc	nc	nb	na	ne	oc	ob	oa	oe	pc	pb	pa	pe	ic	ib	ia	ie	lc	lb	la	le	mc	mb	ma	me
oe	oe	oa	ob	OC.	pe	pa	pb	pc	ne	na	nb	nc	me	ma	mb	mc	ie	ia	ib	ic	le	la	lb	lc
oa	oa	oe	0C	ob	pa	pe	pc	pb	na	ne	nc	nb	ma	me	mc	mb	ia	ie	ic	ib	la	le	lc	lb
ob	ob	0C	oe	oa	pb	pc	pe	pa	nb	nc	ne	na	mb	mc	me	ma	ib	ic	ie	ia	lb	lc	le	la
0C	0C	ob	oa	oe	pc	pb	pa	pe	nc	nb	na	ne	mc	mb	ma	me	ic	ib	ia	ie	lc	lb	la	le
pe	pe	pa	pb	pc	ne	na	nb	nc	oe	oa	ob	00	le	la	lb	lc	me	ma	mb	mc	ie	ia	ib	ic
pa	pa	pe	pc	pb	na	ne	nc	nb	oa	oe	0C	ob	la	le	lc	lb	ma	me	mc	mb	ia	ie	ic	ib
pb	pb	pc	pe	pa	nb	nc	ne	na	ob	0C	oe	oa	lb	lc	le	la	mb	mc	me	ma	ib	ic	ie	ia
pc	pc	pb	pa	pe	nc	nb	na	ne	OC.	ob	oa	oe	lc	lb	la	le	mc	mb	ma	me	ic	ib	ia	ie

It is easy to see that

 $\eta(m,b) \ge \phi(m_1,m_2) \times \psi(b_1,b_2)$ for all $m=m_1m_2$ and $b=b_1b_2$ in G and H respectively, also that $\eta(m,b) \ge \eta(m^{-1},b^{-1})$.

It is a routine to see that

$$\begin{split} &\eta(g,h) \geq \Lambda\{\varnothing(g_1), \varnothing(g_2)\} \times \Lambda\{\psi(h_1), \psi(h_2)\}\\ \text{for every } &g=g_1g_2, \ h=h_1h_2 \ \text{in } G \ \text{and} \ H \ \text{and} \\ &\eta(g,h) \geq \eta(g^{-1},h^{-1})\,. \end{split}$$

Hence, $\eta:G\times H\to (0,1)$ is a fuzzy bigroup of the bigroup G and H

Theorem (3.7): Let α and β be fuzzy subgroups of G and H respectively such that $\alpha(g) \leq \phi(g)$ and $\beta(h) \leq \psi(h)$ for every $(g,h) \in G \times H$. Let $\eta: G \times H \to (0,1)$ be a fuzzy bigroup of $G \times H$ then;

- (i) $\rho \leq \eta$ for every $g, h \in G \times H$,
- (ii) the fuzzy function $\rho = \alpha \times \beta : G \times H \to (0,1) \text{ is also a}$ fuzzy bigroup of the bigroup $G \times H$.

Proof: (i) Since $\alpha(g) \leq \phi(g)$ and $\beta(h) \leq \psi(h)$ for every $(g,h) \in G \times H$, then $\Lambda\{\emptyset(g_1), \emptyset(g_2)\} \times \Lambda\{\psi(h_1), \psi(h_2)\} \leq \Lambda\{\alpha(g_1), \alpha(g_2)\} \times \Lambda\{\beta(h_1), \beta(h_2)\}.$

Therefore $\alpha \times \beta \leq \emptyset \times \psi(h)$. This implies that $\rho \leq \eta$.

Definition (3.8): Let $\eta: G \times H \to (0,1)$ be a fuzzy bigroup. The bilevel subset of the fuzzy bigroup η denoted by η_t is defined as $\eta_t \coloneqq \{(g,h) \in G \times H : \eta(g,h) \ge t\}$

for $t \in [0,1]$.

The following proposition is a direct consequence of Definition 3.8 and the conventional definition of level subset of a set.

Proposition (3.9): Let ϕ_t be a t- level subset of the group G and ψ_s be a s- level subset of the group H, then:

- (i) η_t is a t- bilevel subset of the bigroup $G \times H$ if $t \le s$
- (ii) η_s is a s-bilevel subset of the bigroup $G \times H$ if $s \le t$.

Proof: (i) For $x \in G$ such that $\phi(x) \ge t$, then $x \in \phi_t$ and for $y \in H$ such that $\psi(y) \ge s$, then $y \in \psi_s$. Therefore, for

$$(x, y) \in \phi_t \times \psi_s \subseteq G \times H, \ \eta(x, y) \ge \min\{\phi(x), \psi(s)\}$$

 $\geq \min\{t, s\} = t.$

Hence, η_t is a bilevel subset of the bigroup $\times H$. The proof of (ii) is similar.

Theorem (3.10): The t- bilevel subset $\eta_t \subseteq \phi_t \times \psi_s$ of the bigroup $G \times H$ is a t-bilevel fuzzy sub bigroup of $G \times H$ if the following conditions are satisfied:

- (i) $0 \le s \le t \le 1$
- (ii) for $t \in \text{Im } \phi$ and $s \in \text{Im } \psi$, ϕ_t and ψ_s are fuzzy subgroups of G and H respectively.
- (iii) $t \leq \min\{\phi(e_1), \psi(e_2)\}$

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where e_1 and e_2 are the identity elements of ${\it G}$ and ${\it H}$ respectively.

Proof:

$$\begin{split} & \eta_t = \{(x,y) \in G \times H: \ \eta(x,y) \geq t\} \\ & \text{i.e.} \ \eta_t = \{(x,y) \in G \times H: \phi \times \psi(x,y) \geq t\} \\ & \text{i.e.} \ \eta_t = \{(x,y) \in G \times H: \phi(x) \geq t, \psi(y) \geq t\} \\ & \text{Since} \ 0 \leq s \leq t \leq 1, \\ & \eta_t = \{(x,y) \in G \times H: \phi(x) \geq t, \psi(y) \geq t\} \\ & \Rightarrow \eta_t = \{(x,y) \in G \times H: \phi(x) \geq t, \ \psi(y) \geq s\} \end{split}$$

Let $x_1, x_2 \in \phi_t$, and since ϕ_t is a fuzzy subgroup of G, then $\emptyset(x_1x_2^{-1}) \geq \Lambda\{\emptyset(x_1), \emptyset(x_2)\}$. Also, for $y_1, y_2 \in \psi_s$, since ψ_s is a fuzzy subgroup of H, then $\psi(y_1y_2^{-1}) \geq \Lambda\{\psi(y_1), \psi(y_2)\}$.

Therefore,

$$\eta(x_1 x_2^{-1}, y_1 y_2^{-1}) = \emptyset(x_1 x_2^{-1}) \times (y_1 y_2^{-1})
\ge \Lambda\{\emptyset(x_1), \emptyset(x_2)\} \times \Lambda\{\emptyset(y_1), \emptyset(y_2)\}.$$

It is easy to infer that $t \leq \min\{\phi(e_1), \psi(e_2)\}$ where e_1 and e_2 are the identity elements of G and G respectively by proposition (3.4). Hence, the result follows.

Conclusion

We have been able to define fuzzy bigroup from the idea of Cartesian product of groups and

construct example. We also discussed some fundamental properties of this fuzzy bigroup.

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